

## ADVANCED PROBABILISTIC METHOD DEVELOPMENT

P.H. Wirsching  
The University of Arizona  
Tucson, Arizona

Advanced structural reliability methods are utilized on the PSAM project to provide a tool for analysis and design of space propulsion system hardware. The role of the effort at the University of Arizona is to provide reliability technology support to this project. Reliability refers, in general, to application of probabilistic and statistical methods to manage uncertainties in engineering design.

PSAM computer programs will provide a design tool for analyzing uncertainty associated with thermal and mechanical loading, material behavior, geometry, and the analysis methods used. Specifically, reliability methods are employed (a) to perform sensitivity analyses (a study of the importance of each design factor), (b) to establish the distribution of a critical response variable, (e.g., stress, deflection), (c) to perform reliability assessment, and (d) ultimately to produce a design which will minimize cost and/or weight (Fig. 1).

Uncertainties in the design factors of space propulsion hardware, as defined in Fig. 1, are described by probability models constructed using statistical analysis of data. Two examples are provided in Fig. 2 and Fig. 3. Turbine blade twist (a geometric variable) has a statistical distribution (Fig. 2), and significant variability in material properties are observed in laboratory tests (Fig. 3). Statistical methods are employed to produce a probability model, i.e., a statistical synthesis or summary of each design variable (e.g., each  $X_i$  in Fig. 4) in a format suitable for reliability analysis and ultimately, design decisions.

Given the probability model for each random variable, and the design equation, i.e., how the variables are related (e.g., through the NESSUS/FEM computer code), two basic methods for computing the distribution of a response variable are available. They are, fast probability integration (FPI), (Fig. 4) and Monte Carlo (Fig. 5). FPI is a complex numerical algorithm which provides "fast" probability calculations, accurate to about  $\pm 5\%$  (See Refs. 1 and 2).

The NESSUS generated perturbed data set, an example of which is shown in Fig. 5, is used as a function of the input variables. Monte Carlo produces an exact solution, but it is expensive to run. There are a large number of different Monte Carlo schemes available; methods of improving efficiency of Monte Carlo, tailored for PSAM application, are being studied. Availability of the response function shown in Fig. 5 is a requirement for performing a fast Monte Carlo analysis. The NESSUS-generated, approximate response function can be used to replace the "exact" simulation models needed by Monte Carlo methods.

The result of PSAM analysis is a distribution of a response variable. But for design purposes, it is often important to be able to specify also the confidence associated with a probability statement. Confidence intervals provide a quantitative description of the quality of the results of the probabilistic analyses (Fig. 6). Efficient methods for constructing confidence intervals are being studied.

In summary, the University of Arizona effort on PSAM is to provide technical support in statistical characterization of design variables, advanced reliability analysis methods, and confidence interval construction (Fig. 7).

#### REFERENCES

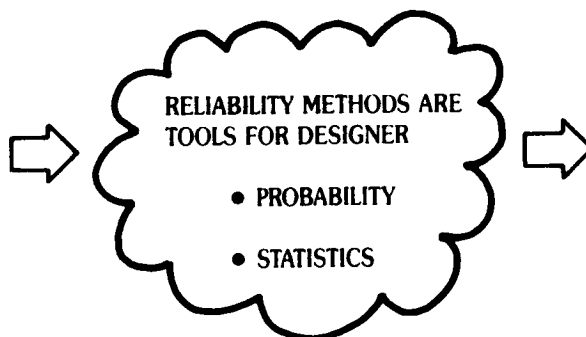
1. Wu, Y.T., "Demonstration of a New Fast Probability Integration Method for Reliability Analysis," Journal of Engineering for Industry, Vol. 109, No. 1, Feb. 1987, pp. 24-28.
2. Wu, Y.T. and Wirsching, P.H., "New Algorithm for Structural Reliability Estimation," to be published in the ASCE Journal of Engineering Mechanics, 1987.

## **RELIABILITY METHODS ARE EFFECTIVE TOOLS FOR DESIGNERS**

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#### SOURCES OF UNCERTAINTY:

- LOADING
- MATERIAL BEHAVIOR
- GEOMETRY
- STRUCTURAL MODELING
- ANALYSIS METHOD
- SMALL SAMPLE SIZE



#### DESIGN APPLICATIONS

- SENSITIVITY ANALYSIS
- VARIABILITY ASSESSMENT
- RELIABILITY ESTIMATION
- COST/WEIGHT REDUCTION
- CONFIDENCE LEVELS

FIGURE 1.

## STATISTICAL METHODS TRANSLATE DATA IN MODELS

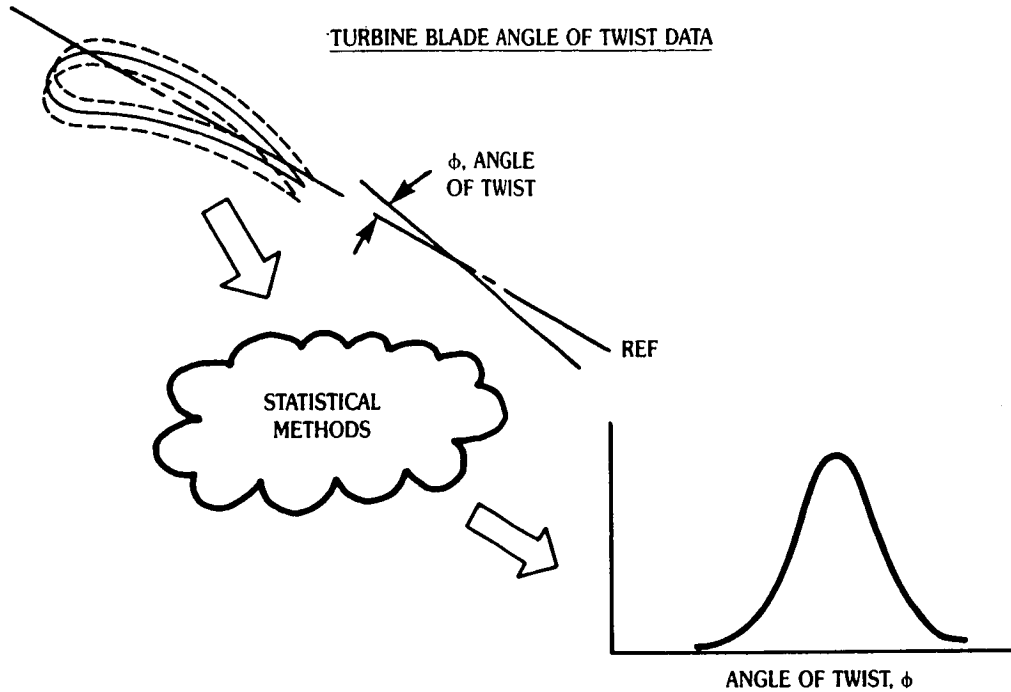


FIGURE 2.

## MATERIAL PROPERTIES EXHIBIT SIGNIFICANT VARIABILITY

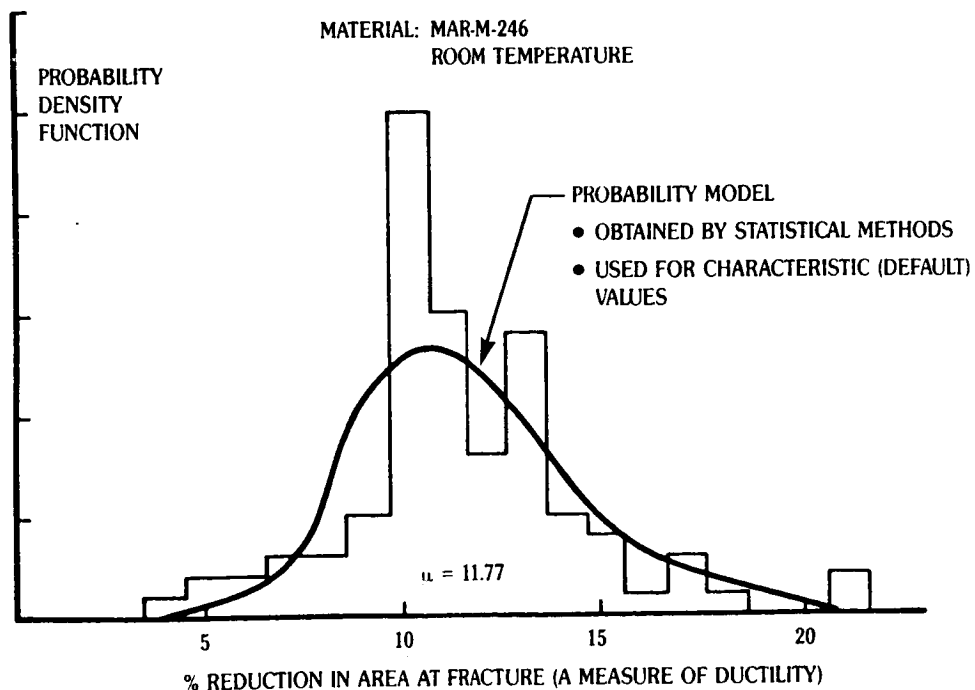


FIGURE 3.

## FPI ALGORITHM IMPROVEMENTS DEMONSTRATED

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- ENHANCED DISTRIBUTION LIBRARY

- MATERIAL BEHAVIOR
- LOADING UNCERTAINTY

- IMPROVED ACCURACY

- VALIDATION PROBLEMS

- HIGHLY NONLINEAR
- MANY VARIABLES

- ACCURACY OF FPI  $\pm 5\%$

- RELATIVE COMPUTER TIME

FPI = 1

MONTE CARLO = 60  
(CONVENTIONAL)

### FATIGUE DAMAGE PREDICTION MODEL

$$D = X_1 + N_0 \left[ \frac{X_2}{X_3 (X_4 X_5)^\gamma} + \frac{1 - X_2}{X_6 (X_4 X_5)^\eta} \right]$$

EACH  $X_i$  HAS DIFFERENT DISTRIBUTION

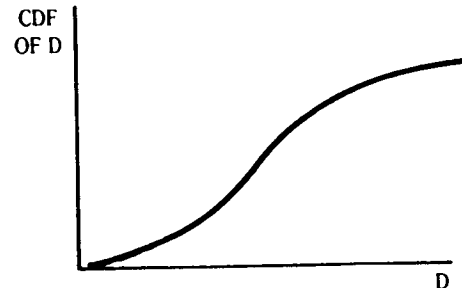
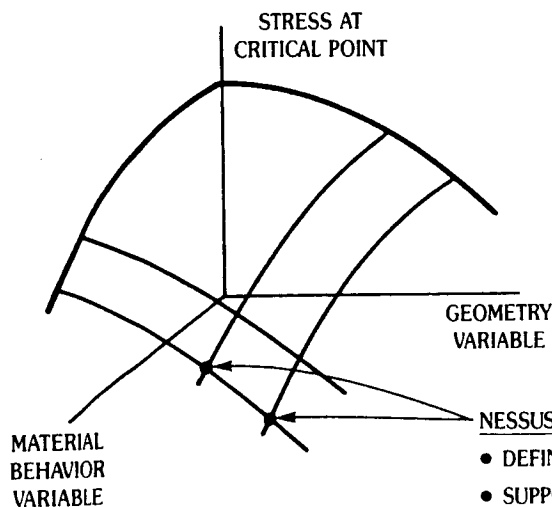


FIGURE 4.

## FAST MONTE CARLO ALGORITHM(S) ESTABLISHED

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- AN INDEPENDENT CHECK ON FPI
- CONVERGES TO EXACT SOLUTION
- MORE EXPENSIVE THAN FPI
- USES NESSUS DATA BASE
- SEVERAL MONTE CARLO METHODS AVAILABLE
- RELATIVE MERITS OF EACH METHOD UNDER STUDY

### NESSUS GENERATED DATA

- DEFINES RESPONSE SURFACE
- SUPPORTS MONTE CARLO SIMULATION

FIGURE 5.

## WHAT IS OUR CONFIDENCE IN THE ANSWER?

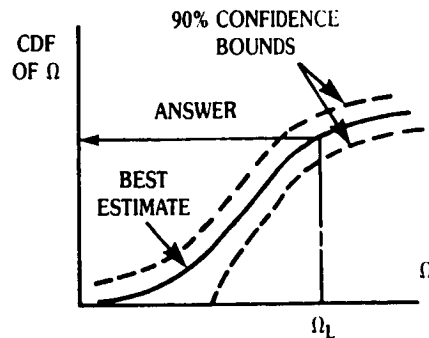
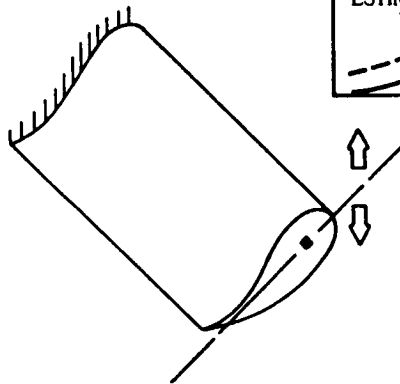
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### PROBLEM:

PROB  $\left[ \text{NATURAL FREQUENCY OF BLADE, } \Omega < \text{FREQUENCY LIMIT, } \Omega_L \right] ?$

### VARIABLES:

- GEOMETRY
- STIFFNESS
- MASS



### CONFIDENCE

- DIFFICULT ANALYSIS PROBLEM
- APPROXIMATE METHOD DEFINED
- USES NESSUS DATA BASE
- GIVES INTERVAL ESTIMATES
  - PROBABILITY
  - VARIABILITY

FIGURE 6.

## UA PROVIDES RELIABILITY TECHNOLOGY SUPPORT

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- LITERATURE REVIEW
- STATISTICAL METHODS TO PRODUCE MODELS
  - GEOMETRY
  - MATERIAL BEHAVIOR
- DISTRIBUTIONAL CHOICES FOR DESIGN VARIABLES AND RESPONSES
- RELIABILITY ASSESSMENT METHODS
  - FPI
  - MONTE CARLO
- CONFIDENCE INTERVALS ON PROBABILITY ESTIMATES

FIGURE 7.